University of Crete
Department of Mathematics
and Applied Mathematics

MEM-227 Field Theory
Instructor: T. Garefalakis
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Midterm Exam

Spring Term 2025-26

The duration of the exam is 100 minutes. The use mobile phones and computing devices is not allowed.

Problem 1 (10 m.)
Compute the degree of the extension $\mathbb{Q}\left(\sqrt[4]{2+\sqrt{6}}\right)/\mathbb{Q}$.

Problem 2 (30 m.)
Let $\omega \in \mathbb{C}$ be a primitive 3rd root of unity.
1. Find $\min(\mathbb{Q}, \omega)$ and a $\mathbb{Q}$-basis of $\mathbb{Q}(\omega)$.
2. Express $\frac{\omega}{\omega+1}$ in terms of the basis you found.
3. Examine if $\mathbb{Q}(\omega)$ is isomorphic to $\mathbb{Q}(i)$, where i is a root of $x^2+1 \in \mathbb{Q}[x]$.

Problem 3 (20 m.)
Prove that $\mathbb{Q}\left(\sqrt[3]{2}\right) \cap \mathbb{Q}\left(\sqrt{3}\right) = \mathbb{Q}$.

Problem 4 (40 m.)
Let $f(x) = (x^2+1)(x^2-2x+3) \in \mathbb{Q}[x]$.
1. Find the splitting field K of f over $\mathbb{Q}$.
2. Compute the degree $[K : \mathbb{Q}]$.
3. Compute the Galois group $\mathrm{Gal}(K/\mathbb{Q})$.
4. Compute $\sigma\left(i+\sqrt{2}+i\sqrt{2}\right)$ for every $\sigma \in \mathrm{Gal}(K/\mathbb{Q})$.