University of Crete
Department of Mathematics
and Applied Mathematics

Sample Midterm Exam

Spring 2025-26

The duration of the exam is 90 minutes. You are not allowed to use any electronic or communications devices.

Problem 1 (10 m.)

Compute the degree of the extension $\mathbb{Q}\left(\sqrt[3]{1+\sqrt{7}}\right)/\mathbb{Q}$.

Problem 2 (30 m.)

Let $f(x) = x^3 - x^2 + 1 \in \mathbb{Q}$.

1. Show that f is irreducible over $\mathbb{Q}$.
2. Let α be a root of f. Find a $\mathbb{Q}$-basis of $\mathbb{Q}(\alpha)$ and write $\frac{1}{\alpha+1}$ in terms of that basis.
3. If $\beta \in \mathbb{Q}(\alpha) \setminus \mathbb{Q}$, show that $\mathbb{Q}(\beta) = \mathbb{Q}(\alpha)$.

Problem 3 (40 m.)

Let $f(x) = x^4 - 2x^3 + 6x^2 - 6x + 9 \in \mathbb{Q}[x]$ and note that $i\sqrt{3}$ is a root.

1. Find the splitting field K of f over $\mathbb{Q}$.
2. Compute the degree $[K : \mathbb{Q}]$.
3. Compute the Galois group $\mathrm{Gal}(K/\mathbb{Q})$.

Problem 4 (20 m.)

Let $f \in F[x]$ be an irreducible polynomial of degree d and K be an extension of F of degree n, and assume that $(n, d) = 1$. Prove that f is irreducible in $K[x]$.